

6. A. A. Tal', "Dynamic properties of single-phase parts of a steam-and-water boiler," *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk*, No. 2, 49-58 (1957).
7. I. A. Kuznetsov, "Unsteady heat transfer in a pipe," *Inzh.-Fiz. Zh.*, 7, No. 11, 16-21 (1964).
8. B. P. Korol'kov, "Transient characteristics of heat exchangers with independent heating," *Izv. Akad. Nauk SSSR, Energ. Transp.*, No. 3, 121-131 (1965).
9. N. N. Smirnova, "Solution of equations of filtration heat transfer by reduction to an equivalent heat conduction equation," in: *Questions of Hydrodynamics and Heat Transfer [in Russian]*, ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1978).
10. Yu. A. Buevich and E. B. Perminov, "Unsteady heating of a stationary granular mass," *Inzh.-Fiz. Zh.*, 38, No. 1, 29-37 (1980).
11. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, Oxford Univ. Press (1959).
12. I. S. Kochenov, "Unsteady flow of fluid in a heated tube," *Dokl. Akad. Nauk SSSR*, 107, No. 5, 689-692 (1956).
13. V. E. Chekalin, "Calculation of Cowper heaters," *Inzh.-Fiz. Zh.*, 3, No. 2, 376-380 (1963).
14. W. Nusselt, "Die Theorie des Winderhitzers," *Z. Verein, Deut. Ing.*, 71, No. 3, 85-91 (1927); W. Nusselt, "Der Beharrungszustang im Winderhitzer," *Z. Verein, Deut. Ing.*, 72, No. 30, 1052-1054 (1928).

NONSTEADY FILTRATION OF SATURATED WATER
VAPOR IN DISPERSE MEDIUM

V. L. Ganzha and G. I. Zhuravskii

UDC 532.546:547.912

Analytic relations are obtained for the calculation of the temperature and pressure distribution in a disperse medium, and also the depth of the heated region in the filtration of saturated water vapor.

Questions associated with the investigation of saturated-water-vapor filtration in disperse media is of particular urgency at present in connection with the prospect of making effective use of the method of vapor-heat treatment as a means of increasing the petroleum yield of a bed. The main aim of the investigation, of course, is to obtain analytical relations allowing the vapor parameters in the course of filtration and its penetration depth in the plate to be obtained.

In most works devoted to the solution of this problem (e.g., [1-4]), integrodifferential heat-balance equations are used. However, in ignoring the hydrodynamics of the process, this approach can obviously only give satisfactory approximation in thermal calculations for very small ΔT , since it is assumed, in the absence of information on the form of the pressure or temperature distribution, that $\Delta T = \text{const}$.

In [5-8], an attempt was made to use relations obtained on the basis of a system of differential equations [9]. However, these equations were derived for the drying of capillary-porous bodies, and cannot be applied outside the scope of problems of diffusional-filtration transfer at small pressure gradients.

The physical picture of the problem is reflected more completely and accurately in [10], where a system of equations of nonisothermal multicomponent filtration is given. However, its use involves serious mathematical difficulties, and moreover mathematical inaccuracy was assumed in deriving the energy equation of the multicomponent flux.

Thus, as far as is known, relations for the calculation of saturated-vapor filtration in a disperse medium which are both sufficiently well-founded and expedient for use are not to be found in the literature at present.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 39, No. 3, pp. 507-512, September, 1980. Original article submitted September 4, 1979.

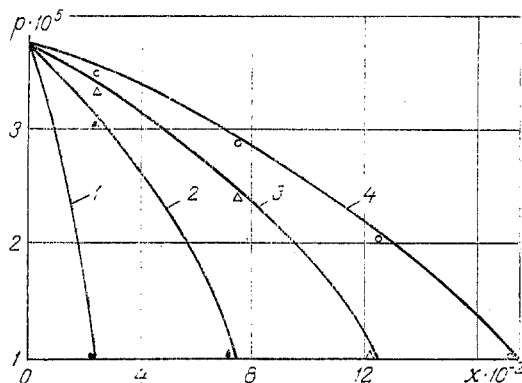


Fig. 1. Comparison of calculated (curves 1-4) and experimental data on the pressure distribution over the layer height at the phase transition boundary, $x = H$; $H = 2.5$ (1), 7.5 (2), 12.5 (3), and 17.5 (4) cm. $P \cdot 10^5$, N/m^2 ; $x \cdot 10^{-2}$ m.

In the deriving analytical relations determining the form of the pressure and temperature distribution in a disperse medium, and also the depth of the heated zone in the filtration of saturated water vapor, a number of assumptions made in earlier experimental work [11] will be adopted. In particular, it is assumed that the compensation process is limited not by heat and mass transfer but by the rate at which vapor reaches the condensation region; that the main flux of phase-transition material is concentrated in a narrow zone at the boundary between the moist (saturated condensate) and dry regions of the disperse medium; that the pressure in the liquid condensate film and the vapor phase is the same; and that vapor filtration behind the condensation front occurs in accordance with Darcy's law.

In considering filtration, the method of steady-state succession is used [12]; in this method, nonsteady filtration is treated as a set of instantaneous steady processes with characteristics continuously changing over time. The Clapeyron-Clausius equation is taken as the relation between the change in pressure at the saturation line and the change in temperature. First, consider the case of linear filtration of water vapor as an ideal gas.

The given problem may be formulated mathematically as follows

$$\frac{dP}{dT} = \frac{r}{T\Delta V}, \quad \rho = \frac{P\mu}{RT}, \quad \rho \frac{dP}{dx} = -\frac{\eta}{k_v} j_v \quad (1)$$

with the boundary condition

$$T_{x=H} = T_H, \quad P_{x=H} = P_0. \quad (2)$$

The solution of Eq. (1) may be written in the form

$$\frac{P^2}{2} \ln C_1 - \left(\frac{P^2}{2} \ln P - \frac{P^2}{4} + C_2 \right) = -\frac{\eta r}{k_v} j_v x \quad (3)$$

or

$$\frac{P^2}{2} \left(\ln \frac{C_1}{P} + \frac{1}{2} \right) = C_2 - \frac{\eta r}{k_v} j_v x, \quad (4)$$

where C_1 and C_2 are constants of integration

$$C_1 = P_0 \exp \left(\frac{\mu r}{RT_H} \right); \quad (5)$$

$$C_2 = \frac{P_0^2}{2} \frac{\mu r}{RT_H} + \frac{P_0^2}{4} + \frac{\eta r}{k_v} j_v H. \quad (6)$$

The expression for determining the temperature distribution is then

TABLE 1. Values of Coefficients

Coeff.	Pressure range P.10 ⁻⁵ N/m ²		
	1-50	50-150	150-200
A	6550	5640	7360
B	4	4	4
C	8,32·10 ²²	1,97·10 ²³	3,35·10 ²³
A'	641	-238	-3900
B'	2	2	2
C'	7,7·10 ⁵	1,46·10 ⁵	362
D ₁	-0,32	-0,572	-1,233
D ₂	-1,54·10 ⁻⁴	-2,98·10 ⁻⁴	-3,97·10 ⁻⁴
D ₃	7,73·10 ⁻⁷	9,05·10 ⁻⁷	3,46·10 ⁻⁷
D ₆	12 459	11 518	18 620

$$\frac{C_1^2}{2} \exp\left(-2 \frac{\mu r}{RT}\right) \left(\frac{\mu r}{RT} + \frac{1}{2}\right) = C_2 - \frac{\eta r}{k_v} j_v x. \quad (7)$$

In deriving Eqs. (4) and (7), it is assumed that the vapor obeys the laws governing an ideal gas; this is sufficiently accurate up to pressures of 1 MPa. At higher pressures, the discrepancy between the behavior of water vapor and an ideal gas becomes very substantial. In this case, according to the Clapeyron-Clausius equation, the relation between the saturated-vapor pressure and temperature may be expressed as [13]

$$P = CT^{-B} \exp\left(-\frac{A}{T}\right) \quad (8)$$

and the equation of state is taken in the form

$$PV = z_0 \frac{m}{\mu} RT, \quad (9)$$

where z_0 , a function of the temperature, may be written, with sufficient accuracy (see below), in the form

$$z_0 = C' \exp\left(-\frac{A'}{T}\right) \frac{1}{T^2}. \quad (10)$$

To simplify the subsequent mathematical manipulations, the structure of this equation is taken to be similar to that of Eq. (8).

For the values of the coefficients in Eqs. (8) and (10) as shown in Table 1, for a range of pressures (from 0.1 to 20 MPa), the maximum discrepancy between the results given by Eqs. (8) and (10) and tabular data [14] does not exceed 3%. Taking Eqs. (8) and (10) into account, Eq. (9) is expressed in the form

$$\rho = \frac{\mu}{C'R} CT^{1-B} \exp\left[(A' - A) \frac{1}{T}\right]. \quad (11)$$

Thus, the system of equations for determining the form of the temperature and pressure distribution for the filtration of a saturated vapor as a real gas takes the form

$$P = CT^{-B} \exp\left(-\frac{A}{T}\right), \quad (12)$$

$$\rho = \frac{\mu}{C'R} CT^{1-B} \exp\left[(A' - A) \frac{1}{T}\right], \quad \rho \frac{dP}{dx} = -\frac{\eta}{k_v} j_v$$

with the boundary conditions $T_{x=H} = T_H$, $P_{x=H} = P_0$. After a series of transformations, and estimating the contribution of individual terms to an accuracy of 0.5%, the solution of the system in Eq. (12) may be reduced to the expression

$$\exp\left[-\frac{(2A - A')}{T}\right] \frac{1}{T^4(2A - A')} \left[\frac{A}{T^3} + \frac{7A}{T^2(2A - A')}\right] + \frac{42A}{T(2A - A')^2} - \frac{B}{T^2} - \frac{6B}{T(2A - A')} - \frac{30B}{(2A - A')^2} \Bigg] = -\frac{\eta j_v C'R}{k_v \mu C^2} x + C_2, \quad (13)$$

where C_2 is determined from the boundary condition at $x=H$, i.e., at $T=T_H$.

Equation (13) describes the temperature distribution at time τ behind the condensation front when the condensation front is a distance H from the vapor inlet to the disperse medium.

Knowing $T=f(x)$, Eq. (8) may be used to find the pressure distribution. The position of the condensation front or the depth of the heated region H can be calculated here from the heat-balance equation but, in contrast to [1-4], the real value of ΔT is taken into account

$$j_v(i''_{in} - i'_C)\tau = \rho_\tau C_\tau \int_0^H \Delta T dx. \quad (14)$$

Since Eq. (13) is the inverse function with respect to $T=f(x)$, Eq. (14) is rewritten in the form

$$j_v \frac{(i''_{in} - i'_C)\tau}{\rho_\tau C_\tau} = \int_{T_H}^{T_1} \varphi(T) dT + H(T_H - T_0), \quad (15)$$

where

$$\begin{aligned} \int_{T_H}^{T_1} \varphi(T) dT = & D_5(T_1 - T_H) + H(T_1 - T_H) + \\ & + \frac{D_4 D_7}{D_6^2} \left[\exp\left(-\frac{D_6}{T_1}\right) \left(\frac{1}{T_1^5} + \frac{5}{D_6 T_1^4} + \frac{20}{D_6^2 T_1^3} \right) - \exp\left(-\frac{D_6}{T_H}\right) \right. \\ & \times \left. \left(\frac{1}{T_H^5} + \frac{5}{D_6 T_H^4} + \frac{20}{D_6^2 T_H^3} \right) \right] - \frac{D_1 D_4}{D_6^2} \left[\exp\left(-\frac{D_6}{T_1}\right) \left(\frac{1}{T_1^4} + \frac{4}{D_6 T_1^3} \right. \right. \\ & \left. \left. + \frac{12}{D_6^2 T_1^2} \right) - \exp\left(-\frac{D_6}{T_H}\right) \left(\frac{1}{T_H^4} + \frac{4}{D_6 T_H^3} + \frac{12}{D_6^2 T_H^2} \right) \right] - \left(\frac{D_2 D_4}{D_6^2} \right. \\ & \times \left[\exp\left(-\frac{D_6}{T_1}\right) \left(\frac{1}{T_1^3} + \frac{3}{D_6 T_1^2} + \frac{6}{D_6^2 T_1} \right) - \exp\left(-\frac{D_6}{T_H}\right) \right. \\ & \times \left. \left(\frac{1}{T_H^3} + \frac{3}{D_6 T_H^2} + \frac{6}{D_6^2 T_H} \right) \right] - \frac{D_3 D_4}{D_6^2} \left[\exp\left(-\frac{D_6}{T_1}\right) \left(\frac{1}{T_1^2} + \frac{2}{D_6 T_1} \right. \right. \\ & \left. \left. + \frac{2}{D_6^2} \right) - \exp\left(-\frac{D_6}{T_H}\right) \left(\frac{1}{T_H^2} + \frac{2}{D_6 T_H} + \frac{2}{D_6^2} \right) \right] \Bigg]. \quad (16) \end{aligned}$$

Estimating the contribution to the total integral of each of the terms in Eq. (16) shows that the last two terms in the curly brackets may be neglected. The resulting error is no more than 0.1%.

After substituting in Eq. (16), Eq. (15) finally yields the expression

$$\begin{aligned} H = & \frac{1}{T_1 - T_0} \left\{ j_v \tau \frac{i''_{in} - i'_C}{\rho_\tau C_\tau} - D_5(T_1 - T_H) - \frac{D_4 D_7}{D_6^2} \left[\exp\left(-\frac{D_6}{T_1}\right) \right. \right. \\ & \times \left. \left(\frac{1}{T_1^5} + \frac{5}{D_6 T_1^4} + \frac{20}{D_6^2 T_1^3} \right) - \exp\left(-\frac{D_6}{T_H}\right) \left(\frac{1}{T_H^5} + \frac{5}{D_6 T_H^4} + \frac{20}{D_6^2 T_H^3} \right) \right] \\ & \left. + \frac{D_1 D_4}{D_6^2} \left[\exp\left(-\frac{D_6}{T_1}\right) \left(\frac{1}{T_1^4} + \frac{4}{D_6 T_1^3} + \frac{12}{D_6^2 T_1^2} \right) - \exp\left(-\frac{D_6}{T_H}\right) \left(\frac{1}{T_H^4} + \frac{4}{D_6 T_H^3} + \frac{12}{D_6^2 T_H^2} \right) \right] \right\}. \quad (17) \end{aligned}$$

The coefficients D_1 - D_7 appearing in Eqs. (16) and (17) are as follows

$$\begin{aligned} D_1 = & \frac{7A}{2A - A'} - B; \quad D_2 = \frac{42A - 6B(2A - A')}{(2A - A')^2}; \\ D_3 = & \frac{30B}{(2A - A')^2}; \quad D_4 = -\frac{k_v \mu C^2}{\eta j_v C' R} = -D_8 \frac{k_v}{\eta j_v}; \\ D_5 = & -\frac{k_v \mu C^2}{\eta j_v C' R} \quad C_2 = D_4 C_2; \quad D_6 = 2A - A'; \quad D_7 = A. \quad (18) \end{aligned}$$

Their values, except for the variable component of the expressions for D_4 and D_5 , are given in Table 1.

The adequacy of the analytical relations obtained in Eqs. (17), (13), (8), (7), and (4) was verified against the experimental data of [11]. The results are shown in Fig. 1, from which it is seen that the agreement between the calculated and experimental data is good. The maximum discrepancy does not exceed 5%, which indicates the reliability of the relations obtained.

NOTATION

P , pressure; T , temperature; r , heat of phase transition; ρ , vapor density; V , vapor specific volume; R , universal gas constant; η , dynamic viscosity; H , coordinate of phase-transition boundary; x , current coordinate; τ , current time; $(i_{in}'' - i_C)$, specific-enthalpy difference between saturated vapor and boiling water at vapor inlet to layer; T_H , temperature at phase-transition boundary; T_0 , initial temperature of medium; j_V , specific mass flow rate of vapor; k_V , permeability of medium; P_0 , initial pressure in disperse medium.

LITERATURE CITED

1. I. W. Mar and R. N. Langenheim, "Reservoir heating by hot fluid injection," *Petrol. Trans. AIME*, 216, 312-315 (1959).
2. B. L. Landrum and J. E. Smith, "Calculation of crude oil recoveries by steam injection," *Petrol. Trans. AIME*, 219, 250-253 (1960).
3. B. T. Willman, V. V. Volleroy, et al., "Laboratory studies of oil recovery by steam injection," *Petrol. Technol.*, 13, 268-272 (1961).
4. B. N. Kazakov and A. Kh. Fatkullin, "Increase in vapor-saturated region of bed in the case of temperature dependent rock heat conduction," in: *Thermal Methods of Working Petroleum Deposits and Treating the Near-Bore Portion of the Bed [in Russian]*, Nedra, Moscow (1971), pp. 69-73.
5. É. M. Simkin and M. A. Bernshtein, "Nonsteady temperature, pressure, and saturation distribution in a bed with vapor heating," in: *Thermal Methods of Working Petroleum Deposits and Treating Near-Bore Portions of Bed [in Russian]*, Nedra, Moscow (1971), pp. 173-180.
6. É. M. Simkin and M. A. Bernshtein, "Temperature, pressure, and saturation with vapor heating in a bed in a semiinfinite plate bounded by tunnels," in: *Thermal Methods of Working Petroleum Deposits and Treating Near-Bore Portions of Bed [in Russian]*, Nedra, Moscow (1971), pp. 307-309.
7. M. L. Surguchev, O. L. Kuznetsov, and É. M. Simkin, *Hydrodynamic, Acoustic and Thermal Cyclic Effects on a Petroleum Bed [in Russian]*, Nedra, Moscow (1975).
8. G. G. Vakhitov, O. L. Kuznetsov, and É. M. Simkin, *Thermodynamics of Near-Bore Region of Petroleum Bed [in Russian]*, Nedra, Moscow (1978).
9. A. V. Lykov and Yu. A. Mikhailov, *Theory of Heat and Mass Transfer [in Russian]*, Gostoptekhizdat, Moscow (1963).
10. M. D. Rozenberg and S. A. Kundin, *Multiphase Multicomponent Filtration in Oil and Gas Extraction [in Russian]*, Nedra, Moscow (1976).
11. V. L. Ganzha and G. I. Zhuravskii, "Experimental investigation of water-vapor filtration in a bed of disperse filling," *Inzh.-Fiz. Zh.*, 38, No. 5 (1980).
12. V. I. Aravin and S. N. Numerov, *Theory of Liquid and Gas Motion in Undeformed Porous Medium [in Russian]*, Moscow (1953).
13. *Thermoengineering Handbook [in Russian]*, Vol. 1, Gostoptekhizdat, Moscow (1956).
14. N. B. Vargaftik, *Tables on the Thermophysical Properties of Liquids and Gases*, Halsted Press (1975).